Rare decay $\pi^0 \to e^+e^-$ constraints on the light CP-odd Higgs in NMSSM

Qin Chang[a,b], Ya-Dong Yang[a,c]

[a]Institute of Particle Physics, Huazhong Normal University, Wuhan, Hubei 430079, P. R. China

[b]Department of Physics, Henan Normal University, Xinxiang, Henan 453007, P. R. China

[c]Key Laboratory of Quark & Lepton Physics, Ministry of Education, P.R. China

May 7, 2009

Abstract

We constrain the light CP-odd Higgs A_1^0 in NMSSM via the rare decay $\pi^0 \to e^+e^-$. It is shown that the possible 3σ discrepancy between theoretical predictions and the recent KTeV measurement of $\mathcal{B}(\pi^0 \to e^+e^-)$ cannot be resolved when the constraints from $\Upsilon \to \gamma A_1^0$, a_μ and $\pi^0 \to \gamma\gamma$ are combined. Furthermore, the combined constraints also exclude the scenario involving $m_{A_1^0} = 214.3$ MeV, which is invoked to explain the anomaly in the $\Sigma^+ \to p\mu^+\mu^-$ decay found by the HyperCP Collaboration.

PACS Numbers: 13.25.Cq, 14.80.Cp

1 Introduction

Theoretically, the rare decay $\pi^0 \to e^+e^-$ starts at the one loop level in the standard model (SM), which has been extensively studied [1, 2, 3, 4, 5, 6, 7, 8, 9, 10] since the first investigation in QED by Drell [1]. It is nontrivial to make precise predictions of the branching ratio $\mathcal{B}_{SM}(\pi^0 \to e^+e^-)$ because its sub-process involves the $\pi^0 \to \gamma^*\gamma^*$ transition form factor. In Refs.[2, 3, 4, 5], the decay was studied via the Vector-Meson Dominance (VMD) approach, where the results are in good agreement with each other and converge in $\mathcal{B}(\pi^0 \to e^+e^-) \sim 6.2 - 6.4 \times 10^{-8}$. By using the measured value of $\mathcal{B}(\eta \to \mu^+\mu^-)$ to fix the counterterms of the chiral amplitude in Chiral Perturbation Theory (ChPT), Savage *et al.* predicted $\mathcal{B}(\pi^0 \to e^+e^-) = (7 \pm 1) \times 10^{-8}$ [6]. Using a procedure similar to that used in Ref.[6] (although with an updated measurement of $\mathcal{B}(\eta \to \mu^+\mu^-)$), Dumm and Pich predicted $(8.3 \pm 0.4) \times 10^{-8}$ [7]. Alternatively, using the lowest meson dominance (LMD) approximation to the large-N_c spectrum of vector meson resonances to fix the counterterms, Knecht *et al.* predicted $(6.2 \pm 0.3) \times 10^{-8}$ [8], which is about 4σ lower than the value predicted by Ref.[7] but which agrees with the others. Most recently, using a dispersive approach to the amplitude and the experimental results of the CELLO [11] and CLEO [12] Collaborations for the pion transition form factor, Dorokhov and Ivanov [9] have found that

$$\mathcal{B}_{SM}(\pi^0 \to e^+e^-) \;=\; (6.23 \,\pm\, 0.09) \times 10^{-8}, \tag{1}$$

which is consistent with most theoretical predictions of $\mathcal{B}_{SM}(\pi^0 \to e^+e^-)$ in the literature. Moreover, their prediction that $\mathcal{B}(\eta \to \mu^+\mu^-) = (5.11 \pm 0.2) \times 10^{-6}$ agrees with the experimental data (which gives a value of $(5.8 \pm 0.8) \times 10^{-6}$ [13]).

Experimentally, the accuracy of the measurements of the decay has increased significantly since the first $\pi^0 \to e^+e^-$ evidence was observed by the Geneva-Saclay group [14] in 1978 with $\mathcal{B}_{SM}(\pi^0 \to e^+e^-) = (22^{+24}_{-11}) \times 10^{-8}$. A detailed summary of the experimental situation can be found in Ref.[15]. Recently, using the complete data set from KTeV E799-II at Fermilab, the KTeV Collaboration has made a precise measurement of the $\pi^0 \to e^+e^-$ branching ratio [16]

$$\mathcal{B}^{no-rad}_{KTeV}(\pi^0 \to e^+e^-) \;=\; (7.48 \,\pm\, 0.29 \,\pm\, 0.25) \times 10^{-8}, \tag{2}$$

after extrapolating the full radiative tail beyond $(m_{e^+e^-}/m_{\pi^0})^2 > 0.95$ and scaling their result back up by the overall radiative correction of 3.4%.

As was already noted in Ref. [9], the SM prediction given in Eq.(1) is 3.3σ lower than the KTeV data. The authors have also compared their result with estimations made by various approaches in the literature and found good agreements. Further analyses have found that QED radiative contributions [17] and mass corrections [18] are at the level of a few percent and are therefore unable to reduce the discrepancy. Although the discrepancy might be due to hadronic dynamics that are as of yet unknown, it is equally possible that this discrepancy is caused by the effects of new physics (NP). In this Letter we will study the latter possibility.

As is known that leptonic decays of pseudoscalar mesons are sensitive to pseudoscalar weak interactions beyond the SM. Precise measurements and calculations of these decays will offer sensitive probes for NP effects at the low energy scale. Of particular interest to us is the rare decay $\pi^0 \to e^+e^-$, which could proceed at tree level via a flavor-conserving process induced by a light pseudoscalar Higgs boson A_1^0 in the next-to-minimal supersymmetric standard model (NMSSM) [19]. We will look for a region of the parameter space of NMSSM that could resolve the aforementioned discrepancy of $\mathcal{B}(\pi^0 \to e^+e^-)$ at 1σ. Then, we combine constraints from a_μ and the recent searches for $\Upsilon(1S), (3S) \to \gamma A_1^0$ by CLEO [20] and BaBar [21], respectively.

2 The amplitude of $\pi^0 \to e^+e^-$ in the SM and the NMSSM

The NMSSM has generated considerable interest in the literature, which extends the minimal supersymmetric SM (MSSM) by introducing a new Higgs singlet chiral superfield $\hat{S}$ to solve the known μ problem in MSSM. The superpotential in the model is [19]

$$W_{NMSSM} = \hat{Q}\hat{H}_u h_u \hat{U}^C + \hat{H}_d \hat{Q} h_d \hat{D}^C + \hat{H}_d \hat{L} h_e \hat{E}^C + \lambda \hat{S}\hat{H}_u \hat{H}_d + \frac{1}{3}\kappa \hat{S}^3, \tag{3}$$

where κ is a dimensionless constant and measures the size of Peccei-Quinn (PQ) symmetry breaking.

In addition to the two charged Higgs bosons, $H^\pm$, the physical NMSSM Higgs sector consists of three scalars h^0, $H_{1,2}^0$ and two pseudoscalars $A_{1,2}^0$. As in the MSSM, $\tan\beta = v_u/v_d$ is the ratio of the Higgs doublet vacuum expectation values $v_u = \langle H_u^0 \rangle = v\sin\beta$ and $v_d = \langle H_d^0 \rangle = v\cos\beta$, where $v = \sqrt{v_d^2 + v_u^2} = \sqrt{2} m_W/g \simeq 174 GeV$. Generally, the masses and singlet contents of the physical fields depend strongly on the parameters of the model (such as, in particular, how well the PQ symmetry is broken). If the PQ symmetry is slightly broken, then A_1^0 can be rather

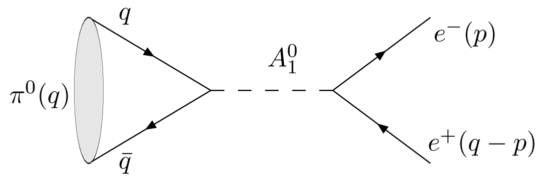

Figure 1: Relevant Feynman diagram within NMSSM.

light, and its mass is given by

$$m^2_{A_1^0} = 3\kappa x A_k + \mathcal{O}(\frac{1}{\tan\beta}) \tag{4}$$

with the vacuum expectation value of the singlet $x = \langle S \rangle$; meanwhile, another pseudoscalar A_2^0 has a mass of order of $m_{H^\pm}$.

For $\pi^0 \to e^+e^-$ decay, the NMSSM contributions are dominated by A_1^0. The couplings of A_1^0 to fermions are [22]

$$\mathcal{L}_{A_i^0 f\bar{f}} = -i\frac{g}{2m_W}\Big(X_d m_d \bar{d}\gamma_5 d + X_u m_u \bar{u}\gamma_5 u + X_\ell m_\ell \bar{\ell}\gamma_5 \ell\Big)A_1^0 \tag{5}$$

where $X_d = X_\ell = \frac{v}{x}\delta_-$ and $X_u = X_d/\tan^2\beta$; thus, the contribution of the $\bar{u}\gamma_5 u A_1^0$ term in $\pi^0 \to e^+e^-$ could be neglected in the large $\tan\beta$ approximation.

To the leading order, the relevant Feynman diagram within NMSSM is shown in Fig. 1. We obtain its amplitude as

$$\mathcal{M}_{A_1^0} = -\frac{G_F}{\sqrt{2}} m_e m^3_{\pi^0} f_{\pi^0} \frac{1}{m^2_{\pi^0} - m^2_{A_1^0}} X_d^2, \tag{6}$$

which is independent of m_d, since m_d in the coupling of $A_1^0 \bar{d}\gamma_5 d$ is canceled by the m_d term of the hadronic matrix

$$\langle 0 \,|\, \bar{d}\gamma_5 d \,|\, \pi^0 \rangle = -\frac{i}{\sqrt{2}} f_{\pi^0} \frac{m^2_{\pi^0}}{2m_d}\,. \tag{7}$$

In the SM, the normalized branching ratio of $\pi^0 \to e^+e^-$ is given by [9]

$$R(\pi^0 \to e^+e^-) = \frac{\mathcal{B}(\pi^0 \to e^+e^-)}{\mathcal{B}(\pi^0 \to \gamma\gamma)} = 2\left(\frac{\alpha_e}{\pi}\frac{m_e}{m_{\pi^0}}\right)^2 \beta_e(m^2_{\pi^0})|\mathcal{A}(m^2_{\pi^0})|^2 \tag{8}$$

where $\beta_e(m^2_{\pi^0}) = \sqrt{1 - 4\frac{m_e^2}{m^2_{\pi^0}}}$ and $\mathcal{A}(m^2_{\pi^0})$ is the reduced amplitude.

To add the NMSSM amplitude to the above amplitudes consistently, we rederive the SM amplitude to look into possible differences between the conventions used in our Letter and the ones used in Ref. [9]. The Feynman diagram that proceeds via two photon intermediate states

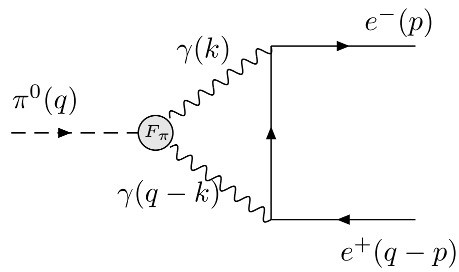

Figure 2: Triangle diagram for $\pi^0 \to e^+e^-$ process.

is shown in Fig. 2. We start with the $\pi^0\gamma^*\gamma^*$ vertex

$$H_{\mu\nu} = -i\, e^2\, \epsilon_{\mu\nu\alpha\beta}\, k^\alpha\, (q-k)^\beta f_{\gamma^*\gamma^*}\, F_{\pi^0\gamma^*\gamma^*}(k^2,(q-k)^2) \tag{9}$$

where k and $q-k$ are the momenta of the two photons, $f_{\gamma^*\gamma^*} = \frac{\sqrt{2}}{4\pi^2\, andf_{\pi^0}}$ is the coupling constant of π^0 to two real photons. $F_{\pi^0\gamma^*\gamma^*}(k^2,(q-k)^2)$ is the transition form factor $\pi^0 \to \gamma^*\gamma^*$, which is normalized to $F_{\pi^0\gamma^*\gamma^*}(0,0) = 1$. The amplitude of Fig. 2 is written as

$$\mathcal{M}_{SM}(\pi^0 \to e^+e^-) = ie^2 \int \frac{d^4k}{(2\pi)^4} \frac{L^{\mu\nu} H_{\mu\nu}}{(k^2+i\varepsilon)\big((k-q)^2+i\varepsilon\big)\big((k-p)^2-m_e+i\varepsilon\big)}, \tag{10}$$

with

$$L^{\mu\nu} \;=\; \bar{u}(p,s)\gamma^\mu(\not{p} - \not{k} + m_e)\gamma^\nu v(q-p,s'). \tag{11}$$

There is a known, convenient way to calculate $L^{\mu\nu}$ with the projection operator for the outgoing e^+e^- pair system[23]

$$\begin{aligned}\mathcal{P}(q-p,p) &= \frac{1}{\sqrt{2}}\big[v(q-p,+)\otimes\bar{u}(p,-) + v(q-p,-)\otimes\bar{u}(p,+)\big] \\ &= \frac{1}{2\sqrt{2t}}\Big[-2m_e q_\mu\gamma^\mu\gamma^5 + \frac{1}{2}\epsilon_{\mu\nu\sigma\tau}\big(p^\sigma(q-p)^\tau - (q-p)^\sigma p^\tau\big)\sigma^{\mu\nu} + t\gamma^5\Big]\end{aligned} \tag{12}$$

where $t = q^2 = m_{\pi^0}^2$. After some calculations, we get

$$\mathcal{M}_{SM}(\pi^0 \to e^+e^-) = 2\sqrt{2}\,\alpha^2\, m_e\, m_{\pi^0}\, f_{\gamma^*\gamma^*}\, A(m_\pi^2) \tag{13}$$

where the reduced amplitude $A(q^2)$ is

$$\mathcal{A}(q^2) = \frac{2\,i}{q^2}\int \frac{d^4k}{\pi^2}\frac{k^2\,q^2\;-\;(q\cdot k)^2}{(k^2+i\varepsilon)\big((k-q)^2+i\varepsilon\big)\big((k-p)^2-m_e+i\varepsilon\big)} F_{\pi^0\gamma^*\gamma^*}(k^2,(q-k)^2). \tag{14}$$

We note that the $\mathcal{A}(q^2)$ derived here is in agreement with Ref. [9]. Further evaluation of the integrals of $\mathcal{A}(q^2)$ is quite subtle and lengthy [2, 24], and only the imaginary part of $\mathcal{A}(m_{\pi^0}^2)$

can be obtained model-independently[1, 2]. In the following calculations, we quote the result of Ref. [9],

$$\mathcal{A}(m_\pi^2) = (10.0 \pm 0.3) - i17.5. \tag{15}$$

With Eq. (6) and Eq. (13), we get the total amplitude

$$\mathcal{M} = 2\sqrt{2}\,\alpha^2\, m_e\, m_{\pi^0}\, f_{\gamma^*\gamma^*}\, A(m_\pi^2) - \frac{G_F}{\sqrt{2}} m_e m_{\pi^0}^3 f_{\pi^0} \frac{1}{m_{\pi^0}^2 - m_{A_1^0}^2} X_d^2. \tag{16}$$

3 Numerical analysis and discussion

Now, we are ready to discuss the effects of A_1^0 numerically, with a focus on the $m_{A_1^0} < 2m_b$ scenarios. The dependence of $\mathcal{B}(\pi^0 \to e^+e^-)$ on the parameter $|X_d|$ is shown in Fig. 3 with $m_{A_1^0} = m_\pi/2$, 214.3MeV, 3GeV as benchmarks. We have used the input parameters $\mathcal{B}(\pi^0 \to \gamma\gamma) = 0.988$ and $f_{\pi^0} = (130.7 \pm 0.4)$ MeV [13]. As shown in Fig. 3, $\mathcal{B}(\pi^0 \to e^+e^-)$ is very sensitive to the parameter $|X_d|$ and $m_{A_1^0}$. For $m_{A_1^0} < m_{\pi^0}$, the NMSSM contribution is deconstructive and reduces $\mathcal{B}(\pi^0 \to e^+e^-)$ at small $|X_d|$ region. For $m_{A_1^0} > m_{\pi^0}$, the NMSSM contribution is constructive and could enhance $\mathcal{B}(\pi^0 \to e^+e^-)$ to be consistent with the KTeV measurement $\mathcal{B}_{KTeV}^{no-rad}(\pi^0 \to e^+e^-) = (7.48 \pm 0.38) \times 10^{-8}$ (where $|X_d|$ strongly depends on $m_{A_1^0}$).

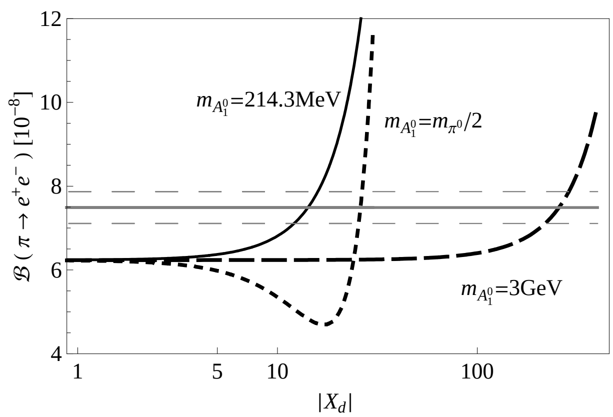

Figure 3: The dependence of $\mathcal{B}(\pi^0 \to e^+e^-)$ on the parameter $|X_d|$ with $m_{A_1^0} = m_{\pi^0}/2$, 214.3 MeV and 3 GeV, respectively. The horizontal lines are the KTeV data, where the solid line is the central value and the dashed ones are the error bars (1σ).

I. Constraint on the scenario of $m_{A_1^0} = 214.3$MeV

It is interesting to note that the HyperCP Collaboration [25] has observed three events for

the decay $\Sigma^+ \to p\mu^+\mu^-$ with a narrow range of dimuon masses. This may indicate that the decay proceeds via a neutral intermediate state, $\Sigma^+ \to pP^0, P^0 \to \mu^+\mu^-$, with a P^0 mass of 214.3 ± 0.5MeV. The possibility of P^0 has been explored in the literature [26, 28, 29, 30]. The authors have proposed A_1^0 as a candidate for the P^0, and have also shown that their explanation could be consistent with the constraints provided by K and B meson decays [26, 27]. It would be worthwhile to check on whether the explanation could be consistent with the $\pi^0 \to e^+e^-$ decay.

Taking $m_{A_1^0} = 214.3$ MeV, we find that $\mathcal{B}(\pi^0 \to e^+e^-)$ is enhanced rapidly and could be consistent with the KTeV data within 1σ for

$$|X_d| = 14.0 \pm 2.4. \tag{17}$$

However, the upper bound $|X_d| < 1.2$ from the a_μ constraint has been derived and used in the calculations of Ref. [26, 29]. So, with the assumption that $m_{A_1^0} = 214.3$ MeV, our result of $|X_d|$ violates the upper bound with a significance of 5σ.

Recently, CLEO [20] and BaBar [21] have searched for the CP-odd Higgs boson in radiative decays of $\Upsilon(1S) \to \gamma A_1^0$ and $\Upsilon(3S) \to \gamma A_1^0$, respectively. For $m_{A_1^0} = 214$MeV, CLEO gives the upper limit

$$\mathcal{B}(\Upsilon(1S) \to \gamma A_1^0) < 2.3 \times 10^{-6} \quad (90\% \; C.L.) \tag{18}$$

which constrains $|X_d| < 0.16$.

The BaBar Collaboration has searched for A_1^0 through $\Upsilon(3S) \to \gamma A_1^0$, $A_1^0 \to$ invisible in the mass range $m_{A_1^0} \leq 7.8$ GeV [21]. From Fig. 5 of Ref. [21], we read

$$\mathcal{B}(\Upsilon(3S) \to \gamma A_1^0) \times \mathcal{B}(A_1^0 \to \text{invisible}) \lesssim 3.5 \times 10^{-6} \quad (90\% \; C.L.) \tag{19}$$

for $m_{A_1^0} = 214$ MeV. Assuming $\mathcal{B}(A_1^0 \to \text{invisible}) \sim 1$, we get the conservative upper limit $|X_d| < 0.19$

All of these upper limits are much lower than the limit of Eq.17 set by $\pi^0 \to e^+e^-$; therefore, the scenario where $m_{A_1^0} \simeq 214$ MeV in NMSSM could be excluded by combining the constraints from $\pi^0 \to e^+e^-$ and the direct searches for Υ radiative decays.

II. Constraints on the parameter space of $m_{A_1^0} - |X_d|$

To show the constraints on NMSSM parameter space from $\pi^0 \to e^+e^-$, we present a scan of $m_{A_1^0} - |X_d|$ space, as shown in Fig. 4. In order to scan the region of $m_{A_1^0} \sim m_{\pi^0}$, the amplitude

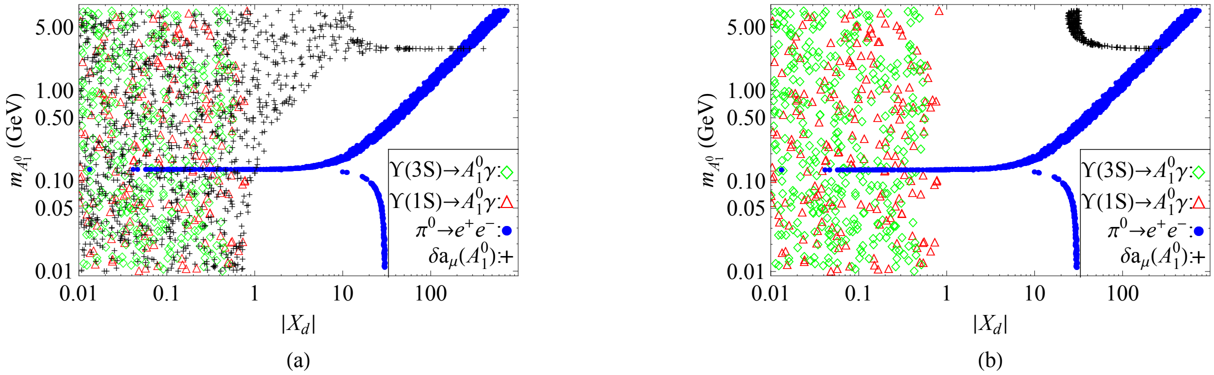

Figure 4: Constraints on the NMSSM parameter space through $\mathcal{B}(\pi^0\to e^+e^-)$, $\mathcal{B}(\Upsilon(1S)\to\gamma A_1^0)$, $\mathcal{B}(\Upsilon(3S)\to\gamma A_1^0)$ and a_μ respectively. The shaded regions are allowed by the labeled processes.

of the A_1^0 contribution in Eq. (6) is replaced by the Breit-Wigner formula

$$\mathcal{M}_{A_1^0} = -\frac{G_F}{\sqrt{2}} m_e m_{\pi^0}^3 f_{\pi^0} \frac{1}{m_{\pi^0}^2 - m_{A_1^0}^2 + i\Gamma(A_1^0) m_{A_1^0}} X_d^2 \,. \tag{20}$$

With the assumption that A_1^0 just decays to electron and photon pairs for $m_{A_1^0} \sim m_{\pi^0}$, the decay width of A_1^0 could be written as

$$\Gamma(A_1^0) = \Gamma(A_1^0 \to e^+e^-) + \Gamma(A_1^0 \to \gamma\gamma) \tag{21}$$

with

$$\begin{aligned} \Gamma(A_1^0 \to e^+e^-) &= \frac{\sqrt{2}G_F}{8\pi} m_e^2 m_{A_1^0} X_d^2 \sqrt{1 - 4\frac{m_e^2}{m_{A_1^0}^2}} \,, \\ \Gamma(A_1^0 \to \gamma\gamma) &= \frac{G_F \alpha^2}{8\sqrt{2}\pi^3} m_{A_1^0}^3 X_d^2 |\sum_i r Q_i^2 k_i F(k_i)|^2, \end{aligned} \tag{22}$$

where $r = 1$ for leptons and $r = N_c$ for quarks, $k_i = m_i^2/m_{A_1^0}^2$ and Q_i is the charge of the fermion in the loop. The loop function $F(k_i)$ reads [31]

$$F(k_i) = \begin{cases} -2\Big(\arcsin\frac{1}{2\sqrt{k_i}}\Big)^2 & \text{for } k_i \geq \frac{1}{4} \,, \\ \frac{1}{2}\Big[\ln\Big(\frac{1+\sqrt{1-4k_i}}{1-\sqrt{1-4k_i}}\Big) + i\pi\Big]^2 & \text{for } k_i < \frac{1}{4} \,. \end{cases}$$

As shown in Fig. 4, only two narrow connected bands of the $|X_d| - m_{A_1^0}$ space survive after the KTeV measurement of $\mathcal{B}(\pi^0\to e^+e^-)$, which show that $\pi^0\to e^+e^-$ is very sensitive to NP scenarios with a light pseudoscalar neutral boson.

In the following, we will determine which part of the remaining parameter space could satisfy the constraints enforced by radiative Υ decays and a_μ simultaneously.

To include the a_μ constraint, we use the experimental result that [32] $a_\mu(Exp) = (11659208.0 \pm 6.3) \times 10^{-10}$ and the SM prediction [33] $a_\mu(SM) = (11659177.8 \pm 6.1) \times 10^{-10}$. The discrepancy is

$$\triangle\, a_\mu = a_\mu(Exp) - a_\mu(SM) = (30.2 \pm 8.8) \times 10^{-10} (3.4\sigma) \tag{23}$$

which is established at a 3.4σ level of significance.

The contributions of A_1^0 to a_μ are given by [34]

$$\begin{aligned}
\delta a_\mu(A_1^0) &= \delta a_\mu^{1-loop}(A_1^0) + \delta a_\mu^{2-loop}(A_1^0)\,, \qquad (24)\\
\delta a_\mu^{1-loop}(A_1^0) &= -\sqrt{2} G_F \frac{m_\mu^2}{8\pi^2} |X_d|^2 f_1(\frac{m_{A_1^0}^2}{m_\mu^2})\,,\\
\delta a_\mu^{2-loop}(A_1^0) &= \sqrt{2} G_F \alpha \frac{m_\mu^2}{8\pi^3} |X_d|^2 \Big[\frac{4}{3}\frac{1}{\tan^2\beta} f_2(\frac{m_t^2}{m_{A_1^0}^2}) + \frac{1}{3} f_2(\frac{m_b^2}{m_{A_1^0}^2}) + f_2(\frac{m_\tau^2}{m_{A_1^0}^2})\Big]
\end{aligned}$$

with

$$\begin{aligned}
f_1(z) &= \int_0^1 dx \frac{x^3}{z(1-x)+x^2}\,, \qquad (25)\\
f_2(z) &= z \int_0^1 dx \frac{1}{x(1-x)-z} \ln \frac{x(1-x)}{z}\,.
\end{aligned}$$

It has been found that the A_1^0 contribution is always negative at the one loop level and worsens the discrepancy in a_μ; however, it could be positive and dominated by the two loop contribution for $A_1^0 > 3$GeV[34]. One should note that there are other contributions to a_μ in NMSSM; for instance, the chargino/sneutino and neutralino/smuon loops. Moreover, the discrepancy $\triangle a_\mu$ could be resolved without pseudoscalars [34]. So, putting a constraint on $|X_d|$ via a_μ is a rather model-dependent process. There are two approximations with different emphases on the role of A_1^0; namely, (i) assuming that $\triangle a_\mu$ is resolved by other contributions and requiring that A_1^0 contributions are smaller than the 1σ error-bar of the experimental measurement, and (ii) assuming that the A_1^0 contributions are solely responsible for $\triangle a_\mu$. In Ref. [26], approximation (i) has been used to derive an upper bound of $|X_d| < 1.2$. We present the a_μ constraints with the two approximations which are shown in Figs. 4(a) and (b), respectively.

From Fig. 4(a), we can find that there are two narrow overlaps between the constraints provided by a_μ and $\mathcal{B}(\pi^0 \to e^+e^-)$: one is for $m_{A_1^0} \sim 3$ GeV with $|X_d| > 150$ and another one is for $m_{A_1^0} \sim 135$ MeV with $|X_d| < 1$.

In the searches for $\Upsilon \to \gamma A_1^0$ decays, CLEO [20] obtains the upper limits for the product of $\mathcal{B}(\Upsilon(1S) \to \gamma A_1^0)$ and $\mathcal{B}(A_1^0 \to \tau^+\tau^-)$ or $\mathcal{B}(A_1^0 \to \mu^+\mu^-)$, while BaBar presents upper limits on $\mathcal{B}(\Upsilon(3S) \to \gamma A_1^0)\times\mathcal{B}(A_1^0 \to invisible)$. All these limits fluctuate with the mass of A_1^0 frequently. For simplicity, we take the loosest upper limit $\mathcal{B}(\Upsilon(1S) \to \gamma A_1^0) \times \mathcal{B}(A_1^0 \to \tau^+\tau^-) < 6 \times 10^{-5}$ of CLEO and assume $\mathcal{B}(A_1^0 \to \tau^+\tau^-) = 1$. Similarly, we also use the loosest upper limits on $\mathcal{B}(\Upsilon(3S) \to \gamma A_1^0) \times \mathcal{B}(A_1^0 \to invisible) < 3.1 \times 10^{-5}$ of BaBar [21] and assume $\mathcal{B}(A_1^0 \to invisible) = 1$. With the loosest upper limits, we get their bounds on the $|X_d| - m_{A_1^0}$ space, which are shown in Fig. 4. From the figure, we can see the bounds (excluding the parameter space $X_d > 1$) for $0 < m_{A_1^0} < 7.8$ GeV. Fig. 4(b) shows that there is no region of parameter space satisfying all the aforementioned constraints if the contribution of A_1^0 is required to solely resolve the a_μ discrepancy.

Of particular interest, as shown in Fig. 4(a), is the parameter space around $m_{A_1^0} \sim 135$ MeV with $|X_d| < 1$ (which is still allowed with approximation (i)). To make a thorough investigation of the space, we read off the upper limits of BaBar [21] from Fig. 5 for the value $m_{A_1^0} \sim 135$ MeV: $\mathcal{B}(\Upsilon(3S) \to \gamma A_1^0) \times \mathcal{B}(A_1^0 \to invisible) \lesssim 3.3 \times 10^{-6}$. With the assumption that $\mathcal{B}(A_1^0 \to invisible) \simeq 1$ and the constraints from $\mathcal{B}(\pi^0{\to}e^+e^-)$, we get

$$|X_d| = 0.10 \pm 0.08, \qquad m_{A_1^0} = 134.99 \pm 0.01\,\text{MeV}, \tag{26}$$

where the constraint on $m_{A_1^0}$ is dominated by $\mathcal{B}(\pi^0{\to}e^+e^-)$ and the limit of $|X_d|$ is dominated by $\mathcal{B}(\Upsilon(3S) \to \gamma A_1^0)$. At first sight, the uncertainties in the abovementioned two parameters are too different. We find that the difference arises from our assumption $\Gamma(A_1^0) \simeq \Gamma(A_1^0 \to e^+e^-) + \Gamma(A_1^0 \to \gamma\gamma)$. From Eqs. (20) and (21), one can see that the X_d^2 factor in $\mathcal{M}_{A_1^0}$ could be canceled out by the one in $\Gamma(A_1^0)$ when $m_{A_1^0}$ approaches m_{π^0}, which results in a very sharp peak for position of $m_{A_1^0}$. Thus, with the well measured quantities given in Eq. (20) and the sensitivity of the peak, $m_{A_1^0}$ turns out to be well-constrained. Furthermore, if we take $m_{A_1^0} = m_{\pi^0}$, we find that X_d^2 is canceled out exactly, so there is no parameter to tune; however, we have $\mathcal{B}(\pi^0 \to e^+e^-) \gg 1$, which violates the unitary bound and is thus excluded.

From the results of Eq.26, we obtain $\delta a_\mu(A_1^0) = (-9.2 \pm 8.9) \times 10^{-12}$ with $\tan\beta = 30$ as a benchmark, which is small enough to be smeared by the chargino/sneutrino and neutralino/smuon contributions. Moreover, we have

$$\Gamma(A_1^0) = (5.7 \pm 5.5) \times 10^{-13}\,\text{MeV}, \tag{27}$$

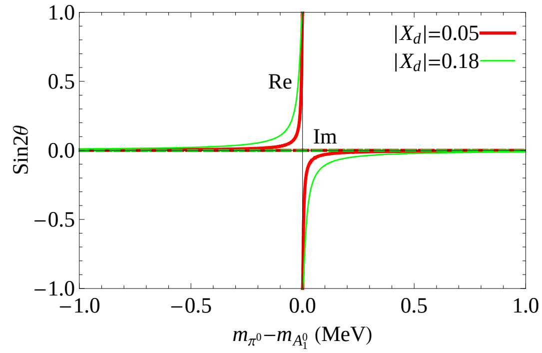

Figure 5: $\sin 2\theta$ versus the mass difference of the unmixed states with $|X_d| = 0.05$ and 0.18. The solid and the dashed lines denote the real and the imaginary parts of $\sin 2\theta$, respectively.

which corresponds to $\tau(A_1^0) \sim 1.2 \times 10^{-9}$ s ($c\tau \sim 36$ cm).

For the case where A_1^0 decays mostly to invisible particles, we take the width of A_1^0 as a free parameter and get $\Gamma(A_1^0) \leq 8.24 \times\ 10^{-6}$GeV, $m_{A_1^0} = 134.99 \pm 0.02$ MeV and $|X_d| \leq 0.18$. In this case, $m_{A_1^0}$ can equal m_{π^0}, and it is found that $\Gamma(A_1^0) \leq 3.3 \times 10^{-6}$ GeV and $|X_d| \leq 0.18$.

III. The resonant effects of $m_{A_1^0} \sim m_{\pi^0}$

So far we have included only the width effects of A_1^0 with the Breit-Wigner formula for the propagator of A_1^0. When the masses of A_1^0 and π^0 are very close, the mixing between the two states could modify the parton level $\pi^0 - A_1^0$ coupling. In a manner analogous to Ref.[35], the mixing can be described by introducing off-diagonal elements in the $A_1^0 - \pi^0$ mass matrix

$$\mathcal{M}^2 = \begin{pmatrix} m^2_{A_1^0} - i m_{A_1^0}\Gamma_{A_1^0} & \delta m^2 \\ \delta m^2 & m^2_{\pi^0} - i m_{\pi^0}\Gamma_{\pi^0} \end{pmatrix} \tag{28}$$

with $\delta m^2 = \sqrt{G_F/4\sqrt{2}} f_{\pi^0} m^2_{\pi^0} X_d$. The complex mixing angle θ between the states is given by

$$\sin^2 2\theta = \frac{(\delta m^2)^2}{\frac{1}{4}(m^2_{A_1^0} - m^2_{\pi^0} - i m_{A_1^0}\Gamma_{A_1^0} + i m_{\pi^0}\Gamma_{\pi^0})^2 + (\delta m^2)^2}. \tag{29}$$

The mass eigenstates $A_1'^0$ and π'^0 are obtained as

$$A_1'^0 = \frac{1}{N}(A_1^0 \cos\theta + \pi^0 \sin\theta), \tag{30}$$

$$\pi'^0 = \frac{1}{N}(-A_1^0 \sin\theta + \pi^0 \cos\theta), \tag{31}$$

where $N = \sqrt{|\sin\theta|^2 + |\cos\theta|^2}$. Then, we can write the decay amplitude of the "physical"

state π'^0 as

$$|\mathcal{M}(\pi'^0 \to e^+e^-)|^2 \;=\; \frac{1}{N^2}\Big(|\cos\theta|^2|\mathcal{M}(\pi^0 \to e^+e^-)|^2 + |\sin\theta|^2|\mathcal{M}(A_1^0 \to e^+e^-)|^2\Big). \qquad (32)$$

Obviously, we obtain the SM result when θ is small.

With $|X_d| = 0.05$ and 0.18, Fig. 5 shows $\sin 2\theta$ as a function of the difference between $m_{A_1^0}$ and m_{π^0}. We note that the imaginary part of $\sin 2\theta$ is negligibly small, since $\Gamma_{A_1^0}m_{A_1^0}+\Gamma_{\pi^0}m_{\pi^0} \ll \delta m^2$. So, the normalization parameter N of the mixing states is nearly unity. Combining the constraints from $\mathcal{B}(\Upsilon(3S) \to \gamma A_1^0)$ and $\mathcal{B}(\pi'^0 \to e^+e^-)$, we get

$$|X_d| = 0.17 \pm 0.01, \qquad m_{A_1^0} \simeq m_{\pi^0}\,. \qquad (33)$$

This confirms the results of our straightforward calculation from Eq. (26), but gives a somewhat stronger constraint on $|X_d|$. With this constraint, we get

$$\Gamma(A_1^0) = (9.8\,\pm\,1.1)\times 10^{-13}\,\mathrm{MeV}, \qquad (34)$$

which is also in agreement with Eq. (27). Furthermore, we get $|\sin\theta|^2 = 0.31 \pm 0.19$.

It is well known that the decay width of $\pi^0 \to \gamma\gamma$ agrees perfectly with the SM prediction, so it is doubtful that that $\pi^0 \to \gamma\gamma$ would be compatible with Higgs with a degenerate mass m_{π^0}. Using the fitted result $|\sin\theta|^2 = 0.31 \pm 0.19$ and

$$|\mathcal{M}(\pi'^0 \to \gamma\gamma)|^2 \;=\; \frac{1}{N^2}\Big(|\cos\theta|^2|\mathcal{M}(\pi^0 \to \gamma\gamma)|^2 + |\sin\theta|^2|\mathcal{M}(A_1^0 \to \gamma\gamma)|^2\Big), \qquad (35)$$

one can easily observe that

$$|\mathcal{M}(A_1^0 \to \gamma\gamma)|^2 \simeq |\mathcal{M}(\pi^0 \to \gamma\gamma)|^2 \qquad (36)$$

is needed to give $\Gamma(\pi' \to \gamma\gamma) \simeq \Gamma(\pi^0 \to \gamma\gamma)$. However, it would require a too large value of $|X_d| \simeq 10^3$; therefore, the degenerate case is excluded.

4 Conclusion

We have studied the decay $\pi^0 \to e^+e^-$ in the NMSSM and shown that it is sensitive to the light CP-odd Higgs boson A_1^0 predicted in the model. The possible discrepancy between the KTeV Collaboration measurement [16] and the theoretical prediction of $\mathcal{B}(\pi^0 \to e^+e^-)$ could

be resolved in NMSSM by the effects of A_1^0 at the tree level. However, it excludes a large fraction of the parameter space of $m_{A_1^0} - |X_d|$. To further constrain the parameter space, we have included bounds from muon $g-2$ and the recent searches for A_1^0 from radiative Υ decays performed at CLEO [20] and BaBar [21]. Combining all these constraints, we have found that

- $\mathcal{B}(\pi^0 \to e^+e^-)$ and $\mathcal{B}(\Upsilon \to \gamma A_1^0)$ put strong constraints on the NMSSM parameter X_d and $m_{A_1^0}$. Due to their different dependences on the two parameters, the interesting scenario where $m_{A_1^0} = 214.3$ MeV is excluded, which would invalidate the A_1^0 hypothesis for the three HyperCP events [25].

- Although these constraints point to a pseudoscalar with $m_{A_1^0} \sim m_{\pi^0}$ and $|X_d| = 0.10 \pm 0.08$ (0.17 ± 0.01, $\pi^0 - A_1^0$ mixing included) in the NMSSM, such an $m_{A_1^0}$ is excluded by $\pi^0 \to \gamma\gamma$ decay.

In this Letter, we have worked in the limit of $X_d \gg X_u$, i.e., the large $\tan\beta$ limit. If we relax the limit and take Eq.5 as a general parameterization of the couplings between a pseudoscalar and fermions, the $\bar{u} - u - A_1^0$ coupling should be included. However, its contribution is deconstructive to the contributions from X_d, since the π^0 flavor structure is $(u\bar{u} - d\bar{d})$. To give a result in agreement with the KTeV Collaboration measurement [16], $X_u \gg X_d$ would be needed, which would imply possible large effects in $\Psi(1S)$ radiative decays. Detailed discussion of this issue would be beyond the main scope of our present study. In summary, we could not find a region of parameter space of NMSSM with $m_{A_1^0} < 7.8$GeV in the large $\tan\beta$ limit that is consistent with the experimental constraints. The HyperCP 214.3MeV resonance and the possible 3.3σ discrepancy in $\pi^0 \to e^+e^-$ decay are still unsolved. Finally, further theoretical investigation is also needed to confirm the discrepancy between the KTeV measurements and SM predications of $\pi^0 \to e^+e^-$ decay. If the discrepancy still persists, it would be an important testing ground for NP scenarios with a light pseudoscalar boson.

The work is supported by the National Science Foundation under contract Nos.10675039 and 10735080.

References

[1] S. Drell, Nuov. Cim. **XI** (1959) 693.

[2] L. Bergström, Zeit. Phys. C **14** (1982) 129.

[3] K.S. Babu and E. Ma, Phys. Lett. B**119** (1982) 449.

[4] L. Bergström, E. Masso, L. Ametller and A. Bramon, Phys. Lett. B **126** (1983) 117.

[5] Ll. Ametller, A. Bramon and E. Massó Phys. Rev. D **48** (1993) 3388 [hep-ph/9302304].

[6] M.J. Savage, M. Luke, and M.B. Wise, Phys. Lett. B**291** (1992) 481.

[7] D. Gómez Dumm and A. Pich Phys. Rev. Lett **80** (1998) 4633 [hep-ph/9801298].

[8] M. Knecht, S. Peris and E. de Rafael Phys. Rev. Lett **83** (1999) 5230 [hep-ph/9908283].

[9] A. E. Dorokhov and M. A. Ivanov, Phys. Rev. D **75** (2007) 114007, arXiv:0704.3498 [hep-ph].

[10] M. D. Scadron and M. Visinescu, Phys. Rev. D **29**, (1984) 911; A. N. Kamal and L. C. Huah, Phys. Rev. D **32**, (1985) 1744; G. Triantaphyllou, Mod. Phys. Lett. A **8**, (1993) 1691 [hep-ph/0301214].

[11] H. J. Behrend *et al.* (CELLO Collaboration), Zeit. Phys. C **49** (1991) 401.

[12] J. Gronberg *et al.* (CLEO Collaboration), Phys. Rev. D **57** (1998) 33 [hep-ex/9707031].

[13] C. Amsler *et al.* (Particle Data Group), Phys. Lett. B **667** (2008) 1.

[14] J. Fischer *et al.*, Phys. Lett. B **73** (1978) 364.

[15] R. Niclasen, Ph.D Thesis, FERMILAB-THESIS-2006-12, UMI-32-07725. http:// lss. fnal. gov/ archive/ thesis/ fermilab-thesis -2006-12. shtml.

[16] E. Abouzaid *et al.*, KTeV Collaboration, Phys. Rev. D **75** (2007) 012004 [hep-ex/0610072].

[17] A. E. Dorokhov and M. A. Ivanov, Eur. Phys. J. C **55** (2008) 193, arXiv:0801.2028 [hep-ph].

[18] A. E. Dorokhov and M. A. Ivanov, JETP Lett. **87** (2008) 531 arXiv:0803.4493 [hep-ph].

[19] H. P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. B **120** (1983) 346; J. M. Frere, D. R. T. Jones and S. Raby, Nucl. Phys. B **222**, (1983) 11; J. P. Derendinger and C. A. Savoy, Nucl. Phys. B **237**, (1984) 307; J. R. Ellis, J. F. Gunion, H. E. Haber, L. Roszkowski and F. Zwirner, Phys. Rev. D **39** (1989) 844; M. Drees, Int. J. Mod. Phys. A **4** (1989) 3635.

[20] W. Love, *et al* (CLEO Collaboration), arXiv:0807.2695 [hep-ex].

[21] B. Aubert, *et al* (BaBar Collaboration), arXiv:0808.0017 [hep-ex].

[22] G. Hiller, Phys. Rev. D **70** (2004) 034018 [hep-ph/0404220].

[23] B. R. Martin, E. De Rafael and J. Smith, Phys. Rev. D **2**, (1970) 179.

[24] M. Pratap and J. Smith, Phys. Rev. D **5**, (1972) 2020; Z. K. Silagadze, Phys. Rev. D **74**, (2006) 054003 [hep-ph/0606284].

[25] H. K. Park, *et al.* (HyperCP Collaboration), Phy. Rev. Lett. **94** (2005) 021801 [hep-ex/0501014].

[26] X. G. He and J. Tandean, Phy. Rev. Lett. **98** (2007) 081802 [hep-ph/0610362].

[27] X. G. He, J. Tandean and G. Valencia, JHEP **0806** (2008) 002, arXiv:0803.4330 [hep-ph].

[28] G. Valencia, arXiv:0805.3285 [hep-ph].

[29] X. G. He, J. Tandean and G. Valencia, Phys. Lett. B **631** (2005) 100 [hep-ph/0509041].

[30] N. G. Deshpande, G. Eilam and J. Jiang, Phys. Lett. B **632** (2006) 212 [hep-ph/0509081].

[31] J. F. Gunion, G. Gamberini and S. F. Novaes, Phys. Rev. D **38**, (1988) 3481.

[32] G. W. Bennett *et al.* (Muon $(g-2)$ Collaboration), Phys. Rev. D **73** (2006) 072003;

[33] M. Passera, W. J. Marciano and A. Sirlin, arXiv:0804.1142.

[34] F. Domingo and U. Ellwanger, JHEP **0807** (2008) 079 arXiv:0806.0733 [hep-ph].

[35] P. J. Franzini and F J. Gilman, Phys. Rev. D **32**, (1985) 237; M. Drees and K. i. Hikasa, Phys. Rev. D **41**, (1990) 1547; E. Fullana, M. A. Sanchis-Lozano, Phys. Lett. B **653** (2007) 67 [hep-ph/0702190].